Solomon Practice Paper

Core Mathematics 4F

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	8	
2	8	
3	9	
4	9	
5	11	
6	13	
7	17	
Total:	75	

How I can achieve better:

-
-
-

Last updated: May 5, 2023

1. A curve has the equation [8]

$$2x^2 + xy - y^2 + 18 = 0.$$

Find the coordinates of the points where the tangent to the curve is parallel to the x-axis.

2. Use the substitution $x = 2\tan(u)$ to show that [8]

$$\int_0^2 \frac{x^2}{x^2+4}\,\mathrm{d}x = \frac{1}{2}(4-\pi).$$

3. (a) Show that [2]

$$\left(1\frac{1}{24}\right)^{-\frac{1}{2}} = k\sqrt{6}$$

where k is rational.

(b) Expand [4]

$$\left(1+\frac{1}{2}x\right)^{-\frac{1}{2}}, \quad |x| < 2,$$

in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

(c) Use your answer to part (b) with $x = \frac{1}{12}$ to find an approximate value for $\sqrt{6}$, giving your answer to 5 decimal places. [3]

Total: 9

4. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = (7\mathbf{j} - 4\mathbf{k}) + s(4\mathbf{i} - 3\mathbf{j} + \mathbf{k}), \quad \text{and} \quad \mathbf{r} = (-7\mathbf{i} + \mathbf{j} + 8\mathbf{k}) + t(-3\mathbf{i} + 2\mathbf{k}),$$

where s and t are scalar parameters.

(a) Show that the two lines intersect and find the position vector of the point where they meet. [5]

(b) Find, in degrees to 1 decimal place, the acute angle between the lines. [4]

Total: 9

5. A curve has parametric equations

$$x = \frac{t}{2-t}, \quad \text{and} \quad y = \frac{1}{1+t}, \qquad -1 < t < 2.$$

(a) Show that [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}\left(\frac{2-t}{1+t}\right)^2.$$

(b) Find an equation for the normal to the curve at the point where $t = 1$. [3]

(c) Show that the cartesian equation of the curve can be written in the form [4]

$$y = \frac{1+x}{1+3x}.$$

Total: 11

6. (a) Find [3]

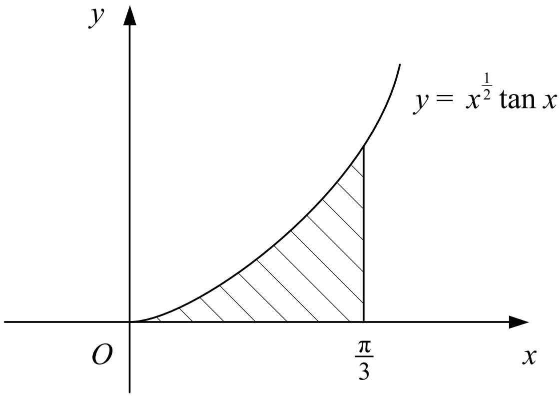
$$\int \tan^2(x)\,\mathrm{d}x.$$

(b) Show that [4]

$$\int \tan(x)\,\mathrm{d}x = \ln|\sec(x)| + c,$$

where c is an arbitrary constant.

Figure shows part of the curve with equation $y = x^{\frac{1}{2}}\tan(x)$.



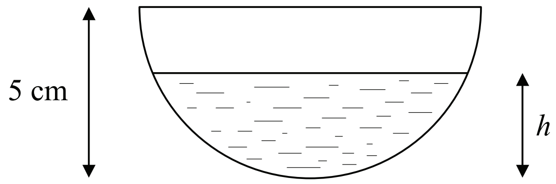
The shaded region bounded by the curve, the x-axis and the line $x = \frac{\pi}{3}$ is rotated through 2π radians about the x-axis.

(c) Show that the volume of the solid formed is [6]

$$\frac{1}{18}\pi^2\left(6\sqrt{3} - \pi\right) - \pi\ln(2).$$

Total: 13

7. Figure shows a hemispherical bowl of radius 5 cm.



The bowl is filled with water but the water leaks from a hole at the base of the bowl. At time t minutes, the depth of water is h cm and the volume of water in the bowl is V cm^3, where

$$V = \frac{1}{3}\pi h^2(15 - h).$$

In a model it is assumed that the rate at which the volume of water in the bowl decreases is proportional to V.

(a) Show that [5]

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{kh(15-h)}{3(10-h)},$$

where k is a positive constant.

(b) Express [3]

$$\frac{3(10-h)}{h(15-h)}$$

in partial fractions.

Given that when $t = 0, h = 5$,

(c) show that [6]

$$h^2(15 - h) = 250\mathrm{e}^{-kt}.$$

Given also that when $t = 2, h = 4$,

(d) find the value of k to 3 significant figures. [3]

Total: 17