Solomon Practice Paper

Core Mathematics 4D

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	8	
3	10	
4	12	
5	12	
6	13	
7	14	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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Last updated: May 5, 2023

1. (a) Find the binomial expansion of $(2-3x)^{-3}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [5]

   (b) State the set of values of x for which your expansion is valid. [1]

Total: 6

2. A curve has the equation

$$x^2 + 3xy - 2y^2 + 17 = 0.$$

   (a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

   (b) Find an equation for the normal to the curve at the point $(3, -2)$. [3]

Total: 8

3. (a) Find the values of the constants A, B, C and D such that [5]

$$\frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \equiv Ax + B + \frac{C}{x} + \frac{D}{x-3}.$$

(b) Evaluate [5]

$$\int_1^2 \frac{2x^3 - 5x^2 + 6}{x^2 - 3x}\,\mathrm{d}x,$$

giving your answer in the form $p + q\ln(2)$, where p and q are integers.

Total: 10

4. A mathematician is selling goods at a car boot sale. She believes that the rate at which she makes sales depends on the length of time since the start of the sale, t hours, and the total value of sales she has made up to that time, £x.

   She uses the model

   $$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k(5-t)}{x},$$

   where k is a constant.

   Given that after two hours she has made sales of £96 in total,

   (a) solve the differential equation and show that she made £72 in the first hour of the sale. [8]

   The mathematician believes that is it not worth staying at the sale once she is making sales at a rate of less than £10 per hour.

   (b) Verify that at 3 hours and 5 minutes after the start of the sale, she should have already left. [4]

Total: 12

5. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -6 \end{pmatrix} + t \begin{pmatrix} 3 \\ a \\ b \end{pmatrix},$$

where a and b are constants and s and t are scalar parameters.

Given that the two lines are perpendicular,

(a) find a linear relationship between a and b. [2]

Given also that the two lines intersect,

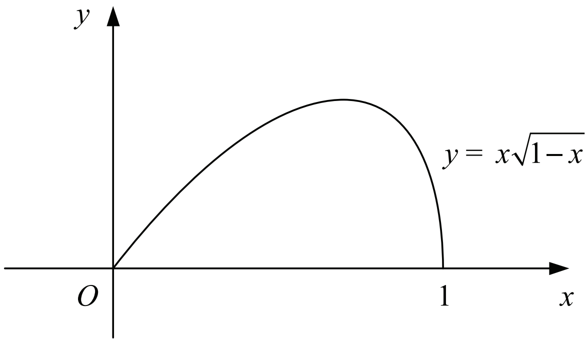
(b) find the values of a and b, [8]

(c) find the coordinates of the point where they intersect. [2]

Total: 12

6. Figure shows the curve with equation

$$y = x\sqrt{1-x}, \qquad 0 \le x \le 1.$$



(a) Use the substitution $u^2 = 1 - x$ to show that the area of the region bounded by the curve and the x-axis is $\frac{4}{15}$. [8]

(b) Find, in terms of π, the volume of the solid formed when the region bounded by the curve and the x-axis is rotated through 360° about the x-axis. [5]

Total: 13

7. A curve has parametric equations

$$x = 3\cos^2(t), \quad \text{and} \quad y = \sin(2t), \qquad 0 \le t < \pi.$$

(a) Show that [4]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{3}\cot(2t).$$

(b) Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. [3]

(c) Show that the tangent to the curve at the point where $t = \frac{\pi}{6}$ has the equation [3]

$$2x + 3\sqrt{3}y = 9.$$

(d) Find a Cartesian equation for the curve in the form $y^2 = \mathrm{f}(x)$. [4]

Total: 14