Solomon Practice Paper

Core Mathematics 4B

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	7	
3	8	
4	9	
5	9	
6	11	
7	12	
8	13	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. Use integration by parts to find [6]

$$\int x^2 \sin(x)\,\mathrm{d}x.$$

2. Given that $y = -2$ when $x = 1$, solve the differential equation [7]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y^2\sqrt{x},$$

giving your answer in the form $y = \mathrm{f}(x)$.

3. A curve has the equation [8]

$$4x^2 - 2xy - y^2 + 11 = 0.$$

Find an equation for the normal to the curve at the point with coordinates $(-1, -3)$.

4. (a) Expand [3]

$$(1+ax)^{-3}, \qquad |ax|<1,$$

in ascending powers of x up to and including the term in x^3. Give each coefficient as simply as possible in terms of the constant a.

Given that the coefficient of x^2 in the expansion of

$$\frac{6-x}{(1+ax)^3}, \qquad |ax|<1,$$

is 3,

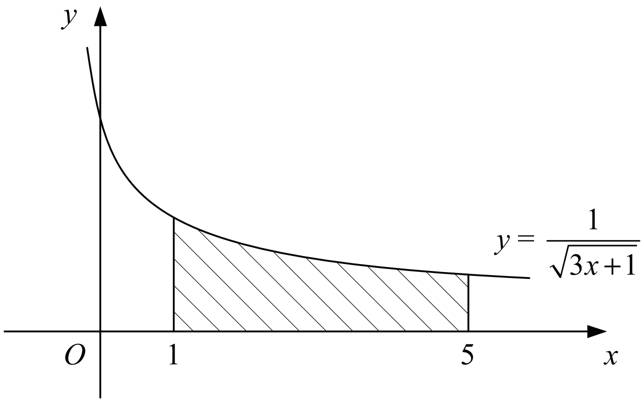
(b) find the two possible values of a. [4]

Given also that $a<0$,

(c) show that the coefficient of x^3 in the expansion of $\dfrac{6-x}{(1+ax)^3}$ is $\frac{14}{9}$. [2]

Total: 9

5. Figure shows the curve with equation $y = \dfrac{1}{\sqrt{3x+1}}$.



The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 5$.

(a) Find the area of the shaded region. [4]

The shaded region is rotated completely about the x-axis.

(b) Find the volume of the solid formed, giving your answer in the form $k\pi \ln(2)$, where k is a simplified fraction. [5]

Total: 9

6.

$$f(x) = \frac{15 - 17x}{(2+x)(1-3x)^2}, \qquad x \neq -2,\ x \neq \frac{1}{3}.$$

(a) Find the values of the constants A, B and C such that [4]

$$f(x) = \frac{A}{2+x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}.$$

(b) Find the value of [7]

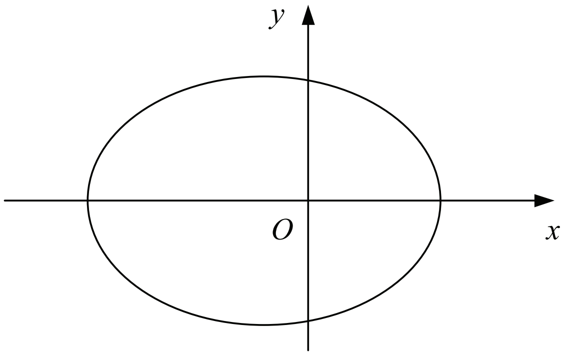
$$\int_{-1}^{0} f(x)\,dx,$$

giving your answer in the form $p + \ln(q)$, where p and q are integers.

Total: 11

7. Figure shows the curve with parametric equations

$$x = -1 + 4\cos(\theta) \quad \text{and} \quad y = 2\sqrt{2}\sin(\theta), \qquad 0 \le \theta < 2\pi.$$



The point P on the curve has coordinates $(1, \sqrt{6})$.

(a) Find the value of θ at P. [2]

(b) Show that the normal to the curve at P passes through the origin. [7]

(c) Find a Cartesian equation for the curve. [3]

Total: 12

8. The line l_1 passes through the points A and B with position vectors $(-3\mathbf{i}+3\mathbf{j}+2\mathbf{k})$ and $(7\mathbf{i}-\mathbf{j}+12\mathbf{k})$ respectively, relative to a fixed origin.

   (a) Find a vector equation for l_1. [2]

   The line l_2 has the equation

$$\mathbf{r} = (5\mathbf{j} - 7\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}).$$

   The point C lies on l_2 and is such that AC is perpendicular to BC.

   (b) Show that one possible position vector for C is $\mathbf{i} + 3\mathbf{j}$ and find the other. [8]

   Assuming that C has position vector $(\mathbf{i} + 3\mathbf{j})$,

   (c) find the area of triangle ABC, giving your answer in the form $k\sqrt{5}$. [3]

Total: 13