Solomon Practice Paper

Core Mathematics 4A

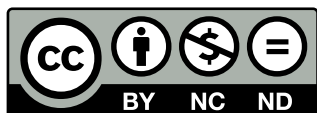# Time allowed: 90 minutes

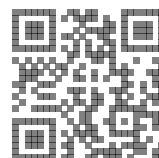**Centre:** www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	11	
4	12	
5	12	
6	12	
7	14	
Total:	75	

How I can achieve better:

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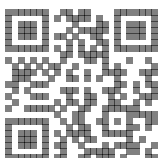1. A curve has the equation   [6]

$$x^2(2+y) - y^2 = 0.$$

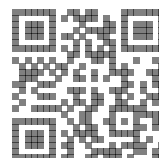Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

2.

$$f(x) = \frac{3}{\sqrt{1-x}}, \quad |x| < 1.$$

(a) Show that $f\left(\frac{1}{10}\right) = \sqrt{10}$. [2]

(b) Expand $f(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [3]

(c) Use your expansion to find an approximate value for $\sqrt{10}$, giving your answer to 8 significant figures. [1]

(d) Find, to 1 significant figure, the percentage error in your answer to part (c). [2]

Total: 8

3. Relative to a fixed origin, O, the line l has the equation

$$\mathbf{r} = (\mathbf{i} + p\mathbf{j} - 5\mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} + q\mathbf{k}),$$

where p and q are constants and λ is a scalar parameter.

Given that the point A with coordinates $(-5, 9, -9)$ lies on l,

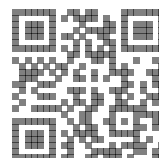(a) find the values of p and q, [3]

(b) show that the point B with coordinates $(25, -1, 11)$ also lies on l. [2]

The point C lies on l and is such that OC is perpendicular to l.

(c) Find the coordinates of C. [4]

(d) Find the ratio $AC : CB$. [2]

Total: 11

4. During a chemical reaction, a compound is being made from two other substances. At time t hours after the start of the reaction, x g of the compound has been produced.
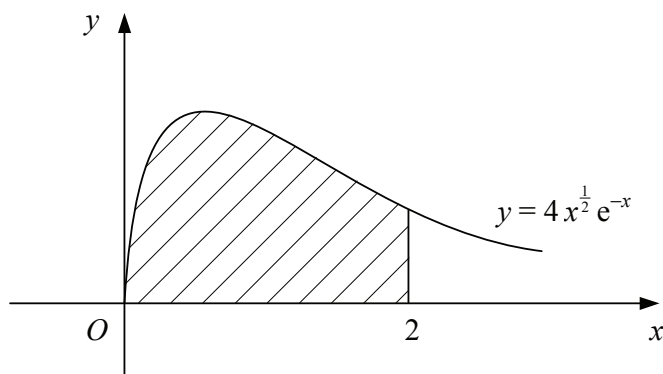
   Assuming that $x = 0$ initially, and that

   $$\frac{\mathrm{d}x}{\mathrm{d}t} = 2(x-6)(x-3),$$

   (a) show that it takes approximately 7 minutes to produce 2 g of the compound.   [10]

   (b) Explain why it is not possible to produce 3 g of the compound.   [2]

   Total: 12
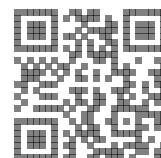
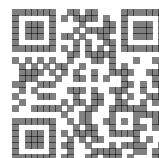5. Figure shows the curve with equation $y = 4x^{\frac{1}{2}}e^{-x}$.

The shaded region is bounded by the curve, the x-axis and the line $x = 2$.

(a) Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region. [5]

The shaded region is rotated through 2π radians about the x-axis.

(b) Find, in terms of π and e, the exact volume of the solid formed. [7]
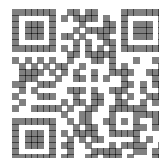
Total: 12

6. (a) Find [4]

$$\int 2\sin(3x)\sin(2x)\,\mathrm{d}x.$$

(b) Use the substitution $u^2 = x + 1$ to evaluate [8]
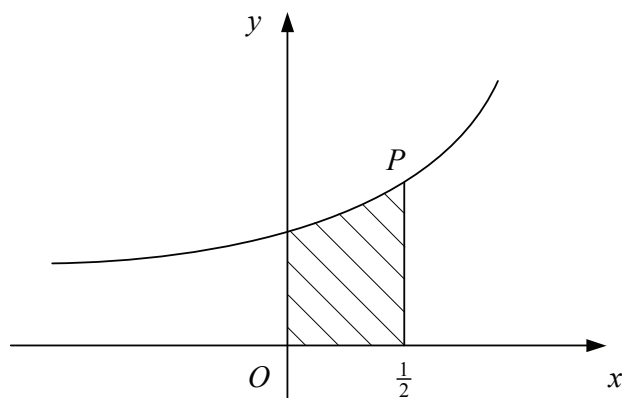
$$\int_0^3 \frac{x^2}{\sqrt{x+1}}\,\mathrm{d}x.$$

Total: 12

7. Figure shows the curve with parametric equations

$$x = \cos(2t) \quad \text{and} \quad y = \csc(t), \qquad 0 < t < \frac{\pi}{2}.$$

The point P on the curve has x-coordinate $\frac{1}{2}$.

(a) Find the value of the parameter t at P. [2]

(b) Show that the tangent to the curve at P has the equation $y = 2x + 1$. [5]

The shaded region is bounded by the curve, the coordinate axes and the line $x = \frac{1}{2}$.

(c) Show that the area of the shaded region is given by [4]

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} k \cos(t) \, dt,$$

where k is a positive integer to be found.

(d) Hence find the exact area of the shaded region. [3]

Total: 14