Solomon Practice Paper

Core Mathematics 3I

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	8	
3	8	
4	8	
5	9	
6	11	
7	12	
8	14	
Total:	75	

How I can achieve better:

-
-
-

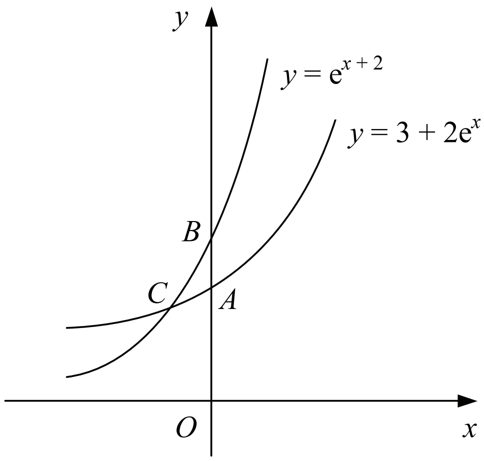
Last updated: May 5, 2023

1. Express [5]

$$\frac{2x}{2x^2+3x-5} \div \frac{x^3}{x^2-x}$$

as a single fraction in its simplest form.

2. Figure shows the curves $y = 3 + 2e^x$ and $y = e^{x+2}$



which cross the y-axis at the points A and B respectively.

(a) Find the exact length AB. [3]

The two curves intersect at the point C.

(b) Find an expression for the x-coordinate of C and show that the y-coordinate of C is $\frac{3e^2}{e^2-2}$. [5]

Total: 8

3.

$$\mathrm{f}(x) = \frac{x^2+3}{4x+1}, x \in \mathbb{R}, x \neq -\frac{1}{4}.$$

(a) Find and simplify an expression for $\mathrm{f}'(x)$. [3]

(b) Find the set of values of x for which $\mathrm{f}(x)$ is increasing. [5]

Total: 8

4. The curve C has the equation $y = x^2 - 5x + 2\ln\left(\frac{x}{3}\right), x > 0$.

   (a) Show that the normal to C at the point where $x = 3$ has the equation [5]

   $$3x + 5y + 21 = 0.$$

   (b) Find the x-coordinates of the stationary points of C. [3]

Total: 8

5. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}(x) &\equiv 6x - 1, && x \in \mathbb{R}, \\ \mathrm{g}(x) &\equiv \log_2(3x+1), && x \in \mathbb{R}, x > -\tfrac{1}{3}. \end{aligned}$$

(a) Evaluate gf(1). [2]

(b) Find an expression for $\mathrm{g}^{-1}(x)$. [3]

(c) Find, in terms of natural logarithms, the solution of the equation $\mathrm{fg}^{-1}(x) = 2$. [4]

Total: 9

6. (a) Use the identities for $\cos(A+B)$ and $\cos(A-B)$ to prove that [4]

$$\cos(P) - \cos(Q) \equiv -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right).$$

(b) Hence find all solutions in the interval $0 \leq x < 180°$ to the equation [7]

$$\cos(5x°) + \sin(3x°) - \cos(x°) = 0.$$

Total: 11

7. The function f is defined by

$$\mathrm{f}(x) \equiv x^2 - 2ax, \quad x \in \mathbb{R},$$

where a is a positive constant.

(a) Showing the coordinates of any points where each graph meets the axes, sketch on separate diagrams the graphs of [6]

i. $y = |\mathrm{f}(x)|$,

ii. $y = \mathrm{f}(|x|)$.

The function g is defined by

$$\mathrm{g}(x) \equiv 3ax, \quad x \in \mathbb{R}.$$

(b) Find $\mathrm{fg}(a)$ in terms of a. [2]

(c) Solve the equation $\mathrm{gf}(x) = 9a^3$. [4]

Total: 12

8.

$$f(x) = 2x + \sin(x) - 3\cos(x).$$

(a) Show that the equation $f(x) = 0$ has a root in the interval $[0.7, 0.8]$. [2]

(b) Find an equation for the tangent to the curve $y = f(x)$ at the point where it crosses the y-axis. [4]

(c) Find the values of the constants a, b and c, where $b > 0$ and $0 < c < \frac{\pi}{2}$, such that [4]

$$f'(x) = a + b\cos(x - c).$$

(d) Hence find the x-coordinates of the stationary points of the curve $y = f(x)$ in the interval $0 \le x \le 2\pi$, giving your answers to 2 decimal places. [4]

Total: 14