Solomon Practice Paper

Core Mathematics 3G

Time allowed: 90 **minutes**

Question	Points	Score
1	7	
2	9	
3	10	
4	10	
5	12	
6	13	
7	14	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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Last updated: May 5, 2023

1. A curve has the equation $y = (3x - 5)^3$.

   (a) Find an equation for the tangent to the curve at the point $P(2, 1)$. [3]

   The tangent to the curve at the point Q is parallel to the tangent at P.

   (b) Find the coordinates of Q. [4]

Total: 7

2. (a) Use the identities for $\cos(A+B)$ and $\cos(A-B)$ to prove that [2]

$$2\cos(A)\cos(B) \equiv \cos(A+B) + \cos(A-B).$$

(b) Hence, or otherwise, find in terms of π the solutions of the equation [7]

$$2\cos\left(x + \frac{\pi}{2}\right) = \sec\left(x + \frac{\pi}{6}\right),$$

for x in the interval $0 \leq x \leq \pi$.

Total: 9

3. Differentiate each of the following with respect to x and simplify your answers.

   (a) $\ln(\cos(x))$ [3]

   (b) $x^2 \sin(3x)$ [3]

   (c) $\dfrac{6}{\sqrt{2x-7}}$ [4]

Total: 10

4. (a) Express $2\sin(x^\circ) - 3\cos(x^\circ)$ in the form $R\sin(x-\alpha)^\circ$ where $R > 0$ and $0 < \alpha < 90$. [4]

   (b) Show that the equation [1]

$$\csc(x^\circ) + 3\cot(x^\circ) = 2$$

   can be written in the form

$$2\sin(x^\circ) - 3\cos(x^\circ) = 1.$$

   (c) Solve the equation [5]

$$\csc(x^\circ) + 3\cot(x^\circ) = 2,$$

   for x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place.

Total: 10

5. (a) Show that $(2x+3)$ is a factor of $(2x^3 - x^2 + 4x + 15)$. [2]

   (b) Hence, simplify [4]

$$\frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15}.$$

   (c) Find the coordinates of the stationary points of the curve with equation [6]

$$y = \frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15}.$$

Total: 12

6. The population in thousands, P, of a town at time t years after 1st January 1980 is modelled by the formula

$$P = 30 + 50e^{0.002t}.$$

Use this model to estimate

(a) the population of the town on 1st January 2010, [2]

(b) the year in which the population first exceeds 84000. [4]

The population in thousands, Q, of another town is modelled by the formula

$$Q = 26 + 50e^{0.003t}.$$

(c) Show that the value of t when $P = Q$ is a solution of the equation [3]

$$t = 1000 \ln \left(1 + 0.08e^{-0.002t}\right).$$

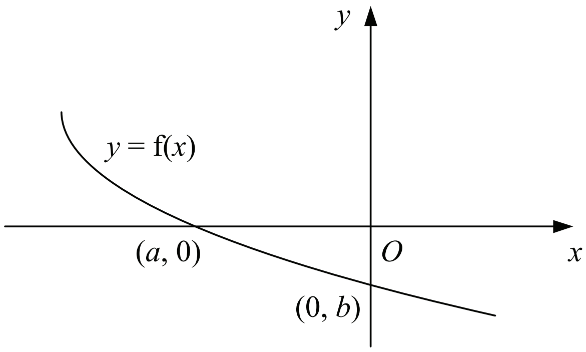
(d) Use the iteration formula [4]

$$t_{n+1} = 1000 \ln \left(1 + 0.08e^{-0.002t_n}\right).$$

with $t_0 = 50$ to find t_1, t_2 and t_3 and hence, the year in which the populations of these two towns will be equal according to these models.

Total: 13

7. Figure shows the graph of $y = \mathrm{f}(x)$ which meets the coordinate axes at the points $(a, 0)$ and $(0, b)$, where a and b are constants.



(a) Showing, in terms of a and b, the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of [6]

i. $y = \mathrm{f}^{-1}(x)$,

ii. $y = 2\mathrm{f}(3x)$.

Given that

$$\mathrm{f}(x) = 2 - \sqrt{x+9}, \quad x \in \mathbb{R}, \quad x \geq -9,$$

(b) find the values of a and b, [3]

(c) find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. [5]

Total: 14