Solomon Practice Paper

Core Mathematics 3F

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	7	
3	8	
4	8	
5	10	
6	11	
7	12	
8	13	
Total:	75	

How I can achieve better:

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1. Solve the equation [6]

$$3\csc(\theta^\circ) + 8\cos(\theta^\circ) = 0$$

for θ in the interval $0 \le \theta \le 180$, giving your answers to 1 decimal place.

2. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}: &\quad x \to 1 - ax, & x \in \mathbb{R}, \\ \mathrm{g}: &\quad x \to x^2 + 2ax + 2, & x \in \mathbb{R}, \end{aligned}$$

where a is a constant.

(a) Find the range of g in terms of a. [3]

Given that $\mathrm{gf}(3) = 7$,

(b) find the two possible values of a. [4]

Total: 7

3. (a) Solve the equation [3]

$$\ln(3x+1) = 2$$

giving your answer in terms of e.

(b) Prove, by counter-example, that the statement [5]

$$\ln\left(3x^2 + 5x + 3\right) \geq 0 \text{ for all real values of } x$$

is false.

Total: 8

4. A curve has the equation $x = y\sqrt{1-2y}$.

   (a) Show that [5]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{1-2y}}{1-3y}.$$

   The point A on the curve has y-coordinate -1.

   (b) Show that the equation of tangent to the curve at A can be written in the form [3]

$$\sqrt{3}x + py + q = 0$$

   where p and q are integers to be found.

Total: 8

5. (a) Sketch the graph of [5]

$$y = 2 + \sec\left(x - \frac{\pi}{6}\right)$$

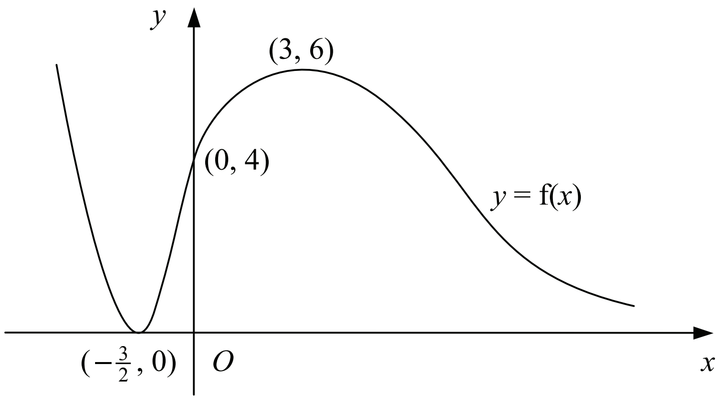
for x in the interval $0 \leq x \leq 2\pi$.

Show on your sketch the coordinates of any turning points and the equations of any asymptotes.

(b) Find, in terms of π, the x-coordinates of the points where the graph crosses the x-axis. [5]

Total: 10

6. Figure shows the curve $y = \mathrm{f}(x)$ which has a minimum point at $\left(-\frac{3}{2}, 0\right)$, a maximum point at $(3, 6)$ and crosses the y-axis at $(0, 4)$.



Sketch each of the following graphs on separate diagrams. In each case, show the coordinates of any turning points and of any points where the graph meets the coordinate axes.

(a) $y = \mathrm{f}(|x|)$ [3]

(b) $y = 2 + \mathrm{f}(x + 3)$ [4]

(c) $y = \frac{1}{2}\mathrm{f}(-x)$ [4]

Total: 11

7.

$$f(x) = 1 + \frac{4x}{2x-5} - \frac{15}{2x^2-7x+5}, x \in \mathbb{R}, x < 1.$$

(a) Show that [5]

$$f(x) = \frac{3x+2}{x-1}.$$

(b) Find an expression for the inverse function $f^{-1}(x)$ and state its domain. [5]

(c) Solve the equation $f(x) = 2$. [2]

Total: 12

8. A curve has the equation $y = x^2 - \sqrt{4 + \ln(x)}$.

   (a) Show that the tangent to the curve at the point where $x = 1$ has the equation [5]

$$7x - 4y = 11.$$

The curve has a stationary point with x-coordinate α.

   (b) Show that $0.3 < \alpha < 0.4$. [3]

   (c) Show that α is a solution of the equation [2]

$$x = \frac{1}{2}\left(4 + \ln(x)\right)^{-\frac{1}{4}}$$

   (d) Use the iteration formula [3]

$$x_{n+1} = \frac{1}{2}\left(4 + \ln(x_n)\right)^{-\frac{1}{4}}$$

   with $x_0 = 0.35$, to find x_1, x_2, x_3 and x_4, giving your answers to 5 decimal places.

Total: 13