Solomon Practice Paper

Core Mathematics 3D

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	5	
2	6	
3	8	
4	9	
5	10	
6	11	
7	12	
8	14	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. The function f is defined by

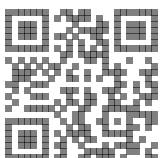$$f(x) \equiv 2 + \ln(3x - 2), \quad x \in \mathbb{R}, \quad x > \frac{2}{3}.$$

   (a) Find the exact value of ff(1). [2]

   (b) Find an expression for $f^{-1}(x)$. [3]

Total: 5
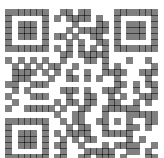
2. Find, to 2 decimal places, the solutions of the equation [6]

$$3\cot^2(x) - 4\csc(x) + \csc^2(x) = 0$$
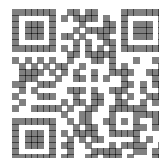
in the interval $0 \leq x \leq 2\pi$.

3. (a) Given that $y = \ln(x)$, find expressions in terms of y for [4]

      i. $\log_2(x)$,

      ii. $\ln\left(\frac{x^2}{e}\right)$.

  (b) Hence, or otherwise, solve the equation [4]

$$\log_2(x) = 4 - \ln\left(\frac{x^2}{e}\right),$$

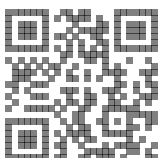   giving your answer to 2 decimal places.

Total: 8

4. (a) Use the identities for $(\sin(A) + \sin(B))$ and $(\cos(A) + \cos(B))$ to prove that [4]

$$\frac{\sin(2x) + \sin(2y)}{\cos(2x) + \cos(2y)} \equiv \tan(x+y).$$

(b) Hence, show that [5]

$$\tan(52.5°) = \sqrt{6} - \sqrt{3} - \sqrt{2} + 2.$$

Total: 9

5.

$$f(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{2x^2 - 5x - 3}, \quad x \in \mathbb{R}, x < -1.$$
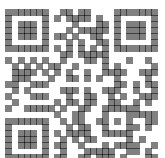
(a) Show that [5]

$$f(x) = \frac{4x-1}{2x+1}.$$

(b) Find an equation for the tangent to the curve $y = f(x)$ at the point where $x = -2$, giving [5]
your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Total: 10

6. A curve has the equation $y = \mathrm{e}^{3x}\cos(2x)$.

   (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

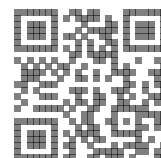   (b) Show that [3]
   $$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \mathrm{e}^{3x}\left(5\cos(2x) - 12\sin(2x)\right).$$

   The curve has a stationary point in the interval $[0, 1]$.

   (c) Find the x-coordinate of the stationary point to 3 significant figures. [4]

   (d) Determine whether the stationary point is a maximum or minimum point and justify your answer. [2]
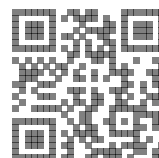
   Total: 11

7. (a) Sketch on the same diagram the graphs of $y = 4a^2 - x^2$ and $y = |2x - a|$, where a is a positive constant. Show, in terms of a, the coordinates of any points where each graph meets the coordinate axes. [6]

   (b) Find the exact solutions of the equation [6]

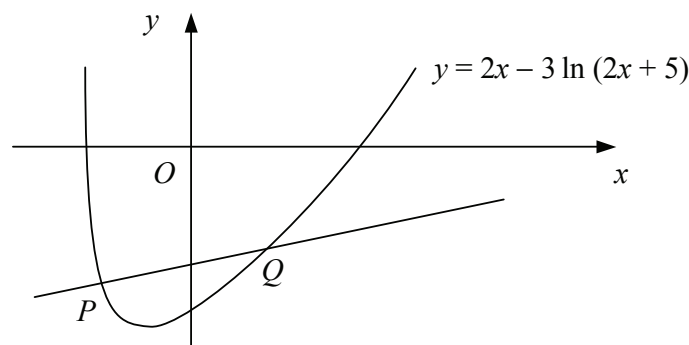 $$4 - x^2 = |2x - 1|.$$

   Total: 12

8. Figure shows the curve with equation $y = 2x - 3\ln(2x+5)$ and the normal to the curve at the point $P(-2, -4)$.

(a) Find an equation for the normal to the curve at P. [4]

The normal to the curve at P intersects the curve again at the point Q with x-coordinate q.

(b) Show that $1 < q < 2$. [3]

(c) Show that q is a solution of the equation [2]

$$x = \frac{12}{7}\ln(2x+5) - 2.$$

(d) Use the iterative formula [5]

$$x_{n+1} = \frac{12}{7}\ln(2x_n + 5) - 2,$$

with $x_0 = 1.5$, to find the value of q to 3 significant figures and justify the accuracy of your answer.

Total: 14