**Solomon Practice Paper**

**Core Mathematics 3A**

**Time allowed:** 90 **minutes**

**Centre:** **www.CasperYC.club**

**Name:**

**Teacher:**

| Question | Points | Score |
|---|---|---|
| 1 | 4 | |
| 2 | 6 | |
| 3 | 8 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 13 | |
| Total: | 75 | |

**How I can achieve better:**

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1. Given that [4]

$$x = \sec^2(y) + \tan(y),$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos^2(y)}{2\tan(y) + 1}.$$

2. The functions f and g are defined by

$$\begin{aligned} \mathrm{f} &: x \to 3x - 4, && x \in \mathbb{R}, \\ \mathrm{g} &: x \to \tfrac{2}{x+3}, && x \in \mathbb{R}, x \neq -3. \end{aligned}$$

   (a) Evaluate fg(1). [2]

   (b) Solve the equation $\mathrm{gf}(x) = 6$. [4]

Total: 6

3. Giving your answers to 2 decimal places, solve the simultaneous equations [8]

$$\begin{aligned} e^{2y} - x + 2 &= 0 \\ \ln(x+3) - 2y - 1 &= 0 \end{aligned}$$

4. (a) Use the derivatives of $\sin(x)$ and $\cos(x)$ to prove that [4]

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan(x) = \sec^2(x).$$

The tangent to the curve $y = 2x\tan(x)$ at the point where $x = \frac{\pi}{4}$ meets the $y$-axis at the point $P$.

(b) Find the $y$-coordinate of $P$ in the form $k\pi^2$ where $k$ is a rational constant. [6]

Total: 10

5. (a) Express [4]

$$3\cos(x^\circ) + \sin(x^\circ)$$

in the form $R\cos(x - \alpha)^\circ$ where $R > 0$ and $0 < \alpha < 90$.

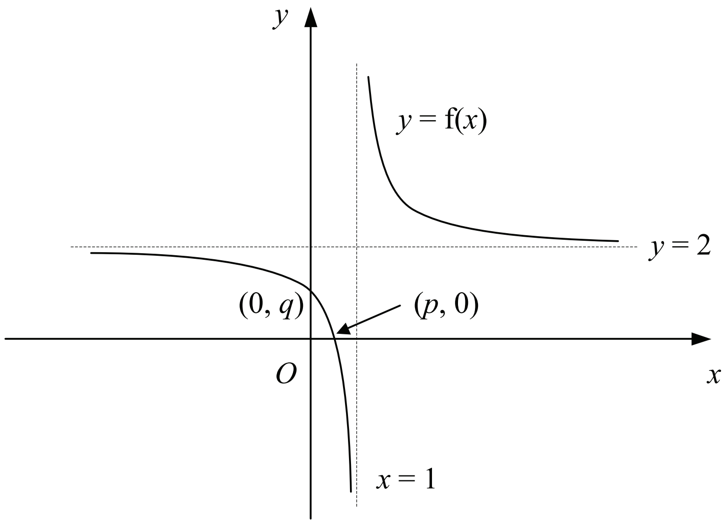
(b) Using your answer to part (a), or otherwise, solve the equation [6]

$$6\cos^2(x^\circ) + \sin(2x^\circ) = 0,$$

for $x$ in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place where appropriate.

Total: 10

6. Figure shows the curve with equation $y = \mathrm{f}(x)$.




The curve crosses the axes at $(p, 0)$ and $(0, q)$ and the lines $x = 1$ and $y = 2$ are asymptotes of the curve.

(a) Showing the coordinates of any points of intersection with the axes and the equations of any asymptotes, sketch on separate diagrams the graphs of [6]

i. $y = |\mathrm{f}(x)|$,

ii. $y = 2\mathrm{f}(x + 1)$.

Given also that

$$\mathrm{f}(x) \equiv \frac{2x-1}{x-1}, \quad x \in \mathbb{R}, x \neq 1,$$

(b) find the values of $p$ and $q$, [3]

(c) find an expression for $\mathrm{f}^{-1}(x)$. [3]

Total: 12

7. (a) i. Show that [6]

$$\sin(x+30)^\circ + \sin(x-30)^\circ \equiv a\sin(x^\circ),$$

where $a$ is a constant to be found.

ii. Hence find the exact value of $\sin(75^\circ) + \sin(15^\circ)$, giving your answer in the form $b\sqrt{6}$.

(b) Solve, for $0 \le y \le 360$, the equation [6]

$$2\cot^2(y^\circ) + 5\csc(y^\circ) + \csc^2(y^\circ) = 0.$$

Total: 12

8.
$$f(x) = \frac{x^4 + x^3 - 5x^2 - 9}{x^2 + x - 6}.$$

(a) Using algebraic division, show that [5]

$$f(x) = x^2 + A + \frac{B}{x + C},$$

where $A, B$ and $C$ are integers to be found.

(b) By sketching two suitable graphs on the same set of axes, show that the equation $f(x) = 0$ has exactly one real root. [3]

(c) Use the iterative formula [5]

$$x_{n+1} = 2 + \frac{1}{x_n^2 + 1},$$

with a suitable starting value to find the root of the equation $f(x) = 0$ correct to 3 significant figures and justify the accuracy of your answer.

Total: 13