Solomon Practice Paper

Core Mathematics 2J

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	6	
3	7	
4	8	
5	8	
6	9	
7	9	
8	9	
9	13	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. During one day, a biological culure is allowed to grow under controlled conditions.

   At 8 a.m. the culture is estimated to contain 20,000 bacteria.

   A model of the growth of the culture assumes that t hours after 8a.m., the number of bacteria present, N, is given by
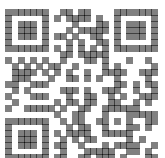
   $$N = 20000 \times (1.06)^t.$$

   Using this model,

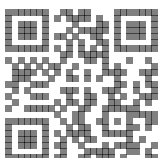   (a) find the number of bacteria present at 11a.m., [2]

   (b) find, to the nearest minute, the time when the initial number of bacteria will have doubled. [4]
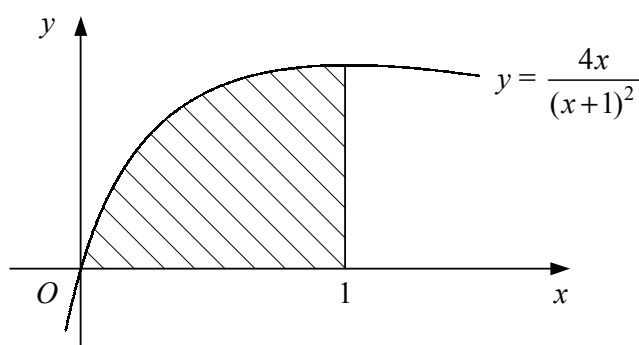
   Total: 6

2. The sides of a triangle have lengths of 7 cm, 8 cm and 10 cm. [6]
   Find the area of the triangle correct to 3 significant figures.

3. Figure shows the curve with equation $y = \frac{4x}{(x+1)^2}$.

The shaded region is bounded by the curve, the x-axis and the line $x = 1$.

(a) Use the trapezium rule with four intervals of equal width to find an estimate for the area of the shaded region. [5]

(b) State, with a reason, whether your answer to part (a) is an under-estimate or an over-estimate of the true area. [2]

Total: 7
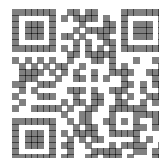
4. The first three terms in the expansion in descending powers of x of

$$\left(x + \frac{k}{x^2}\right)^{15},$$

where k is a constant, are
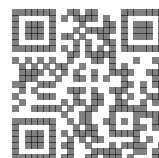
$$x^{15} + 30x^{12} + Ax^9.$$
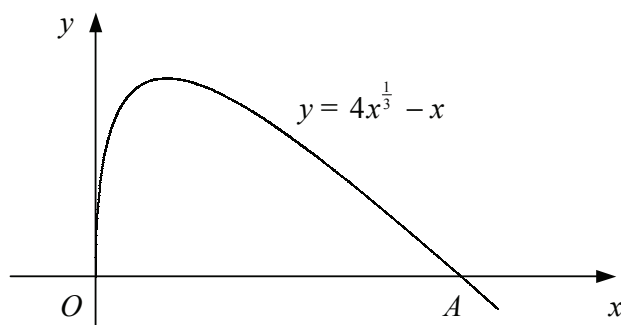
(a) Find the values of k and A. [5]

(b) Find the value of the term independent of x in the expansion. [3]

Total: 8

5. Figure shows the curve with equation $y = 4x^{\frac{1}{3}} - x$, $x \geq 0$.

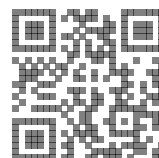The curve meets the x-axis at the origin and at the point A with coordinates $(a, 0)$.

(a) Show that $a = 8$. [3]

(b) Find the area of the finite region bounded by the curve and the positive x-axis. [5]
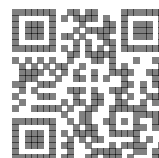
Total: 8

6.

$$\text{f}(x) = \cos(2x),\ 0 \leq x \leq \pi.$$

(a) Sketch the curve $y = \text{f}(x)$. [2]

(b) Write down the coordinates of any points where the curve $y = \text{f}(x)$ meets the coordinate axes. [3]

(c) Solve the equation $\text{f}(x) = 0.5$, giving your answers in terms of π. [4]

Total: 9

7. The points P and Q have coordinates $(-2, 6)$ and $(4, -1)$ respectively.

    Given that PQ is a diameter of circle C,

    (a) find the coordinates of the centre of C, [2]

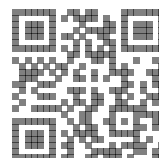   (b) show that C has the equation [5]

    $$x^2 + y^2 - 2x - 5y - 14 = 0.$$

    The point R has coordinates $(2, 7)$.

    (c) Show that R lies on C and hence, state the size of $\angle PRQ$ in degrees. [2]

    Total: 9

8. The second and third terms of a geometric series are $\log_3(4)$ and $\log_3(16)$ respectively.

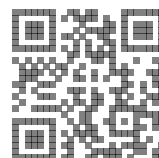   (a) Find the common ratio of the series. [3]

   (b) Show that the first term of the series is $\log_3(2)$. [2]

   (c) Find, to 3 significant figures, the sum of the first six terms of the series. [4]

   Total: 9

9.

$$f(x) = x^3 - 4x^2 - 3x + 18.$$

(a) Show that $(x - 3)$ is a factor of f(x). [2]

(b) Fully factorise f(x). [4]

(c) Using your answer to part (b), write down the coordinates of one of the turning points of the curve $y =$ f(x) and give a reason for your answer. [2]

(d) Using differentiation, find the x-coordinate of the other turning point of the curve $y =$ f(x). [5]

Total: 13