Solomon Practice Paper

Core Mathematics 1E

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	4	
2	4	
3	5	
4	6	
5	7	
6	8	
7	9	
8	10	
9	10	
10	12	
Total:	75	

How I can achieve better:

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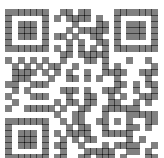
Last updated: May 5, 2023

1. (a) Express $\frac{18}{\sqrt{3}}$ in the form $k\sqrt{3}$. [2]

   (b) Express $(1-\sqrt{3})(4-2\sqrt{3})$ in the form $a+b\sqrt{3}$ where a and b are integers. [2]

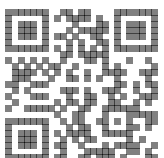   Total: 4

2. Solve the equation [4]

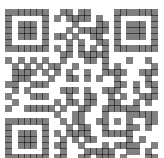$$3x - \frac{5}{x} = 2.$$

3. The straight line l has the equation $x - 5y = 7$.   [5]

   The straight line m is perpendicular to l and passes through the point $(-4, 1)$.

   Find an equation for m in the form $y = mx + c$.

4. A sequence of terms is defined by

$$u_n = 3^n - 2, \quad n \geq 1.$$

(a) Write down the first four terms of the sequence. [2]

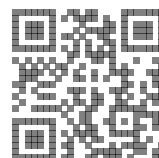The same sequence can also be defined by the recurrence relation

$$u_{n+1} = au_n + b, \quad n \geq 1, \quad u_1 = 1,$$
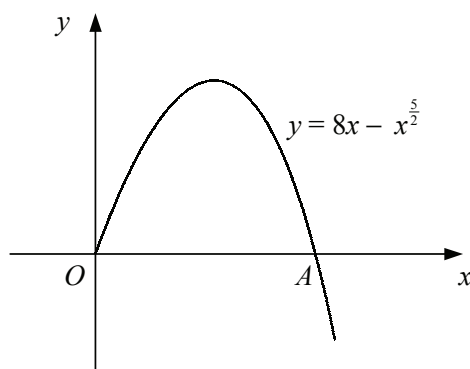
where a and b are constants.

(b) Find the values of a and b. [4]

Total: 6

5. Figure shows the curve with equation $y = 8x - x^{\frac{5}{2}}, \quad x \geq 0$.

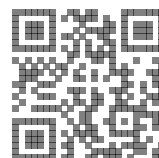The curve meets the x-axis at the origin, O, and at the point A.

(a) Find the x-coordinate of A. [3]

(b) Find the gradient of the tangent to the curve at A. [4]
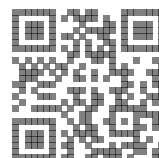
Total: 7

6.

$$f(x) = 2x^2 - 4x + 1.$$

(a) Find the values of the constants a, b and c such that [4]

$$f(x) = a(x + b)^2 + c.$$

(b) State the equation of the line of symmetry of the curve $y = f(x)$. [1]

(c) Solve the equation $f(x) = 3$, giving your answers in exact form. [3]

Total: 8

7.

$$f(x) \equiv \frac{(x-4)^2}{2x^{\frac{1}{2}}}, \quad x > 0.$$
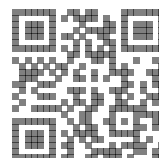
(a) Find the values of the constants A, B and C such that [3]

$$f(x) = Ax^{\frac{3}{2}} + Bx^{\frac{1}{2}} + Cx^{-\frac{1}{2}}.$$

(b) Show that [6]

$$f'(x) = \frac{(3x+4)(x-4)}{4x^{\frac{3}{2}}}$$

Total: 9

8. (a) Describe fully the single transformation that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(x - 1)$. [2]

   (b) Showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes, sketch the graph of $y = \dfrac{1}{x - 1}$. [3]

   (c) Find the x-coordinates of any points where the graph of $y = \dfrac{1}{x - 1}$ intersects the graph of $y = 2 + \dfrac{1}{x}$. Give your answers in the form $a + b\sqrt{3}$, where a and b are rational. [5]

   Total: 10
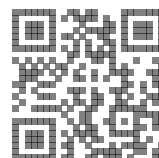
9. A store begins to stock a new range of DVD players and achieves sales of £1500 of these products during the first month.

   In a model it is assumed that sales will decrease by £x in each subsequent month, so that sales of £$(1500 - x)$ and £$(1500 - 2x)$ will be achieved in the second and third months respectively.

   Given that sales total £8100 during the first six months, use the model to

   (a) find the value of x, [4]

   (b) find the expected value of sales in the eighth month, [2]

   (c) show that the expected total of sales in pounds during the first n months is given by $kn(51 - n)$, where k is an integer to be found. [3]

   (d) Explain why this model cannot be valid over a long period of time. [1]

   Total: 10
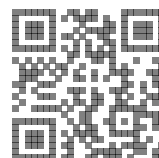
10. The curve C with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 4x + k,$$

where k is a constant.

Given that C passes through the points $(0, -2)$ and $(2, 18)$,

(a) show that $k = 2$ and find an equation for C, [7]

(b) show that the line with equation $y = x - 2$ is a tangent to C and find the coordinates of the point of contact. [5]

Total: 12