**Pearson Edexcel**

**A Level Mathematics 9MA0**

**Unit Test**

**9 Numerical Methods**

**Time allowed:** 50 **minutes**

**School:**

**Name:**

**Teacher:**

| Question | Points | Score |
|---|---|---|
| 1 | 7 | |
| 2 | 5 | |
| 3 | 7 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 11 | |
| Total: | 50 | |

1. $f(x) = x^4 - 8x^2 + 2$.

   (a) Show that the equation $f(x) = 0$ can be written as [2]

   $$x = \sqrt{ax^4 + b}, x > 0$$

   where $a$ and $b$ are constants to be found.

   (b) Let $x_0 = 1.5$. Use the iteration formula [2]

   $$x_{n+1} = \sqrt{ax_n^4 + b}$$

   together with your values of $a$ and $b$ from part (a), to find, to 4 decimal places, the values of $x_1, x_2, x_3$ and $x_4$.

   A root of $f(x) = 0$ is $\alpha$.

   (c) By choosing a suitable interval, prove that $\alpha = -2.782$ to 3 decimal places. [3]

Total: 7

2.

$$g(x) = 3\sin\left(\frac{x}{6}\right)^3 - \frac{1}{10}x - 1, -40 < x < 20,$$

$x$ is in radians.

(a) Show that the equation $g(x) = 0$ can be written as [3]

$$x = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)}\right)$$

(b) Using the formula [2]

$$x_{n+1} = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x_n\right)}\right),$$

with $x_0 = 4$, find to 3 decimal places, the values of $x_1, x_2$ and $x_3$.

Total: 5

3. $f(x) = 2 - 3\sin^3(x) - \cos(x)$, where $x$ is in radians.

   (a) Show that $f(x) = 0$ has a root $\alpha$ between $x = 1.9$ and $x = 2.0$. [2]

   (b) Using $x_0 = 1.95$ as a first approximation, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to $\alpha$, giving your answer to 3 decimal places. [5]

Total: 7

4. $g(x) = \frac{2}{x-1} - \mathrm{e}^x$

   (a) By drawing an appropriate sketch, show that there is only one solution to the equation $g(x) = 0$. [2]

   (b) Show that the equation $g(x) = 0$ may be written in the form $x = 2\mathrm{e}^{-x} + 1$ [2]

   (c) Let $x_0 = 1.5$. [2]

   Use the iterative formula to find to 4 decimal places the values of $x_1, x_2, x_3$ and $x_4$.

   (d) Using $x_0 = 1.5$ as a first approximation, apply the Newton-Raphson procedure once to $g(x)$ to find a second approximation to $\alpha$, giving your answer to 4 decimal places. [4]

Total: 10

5.

$$h(t) = 40\ln(t+1) + \sin\left(\frac{t}{5}\right) - \frac{1}{4}t^2, \quad t \geq 0$$

The graph $y = h(t)$ models the height of a rocket $t$ seconds after launch.

(a) Show that the rocket returns to the ground between 19.3 and 19.4 seconds after launch. [2]

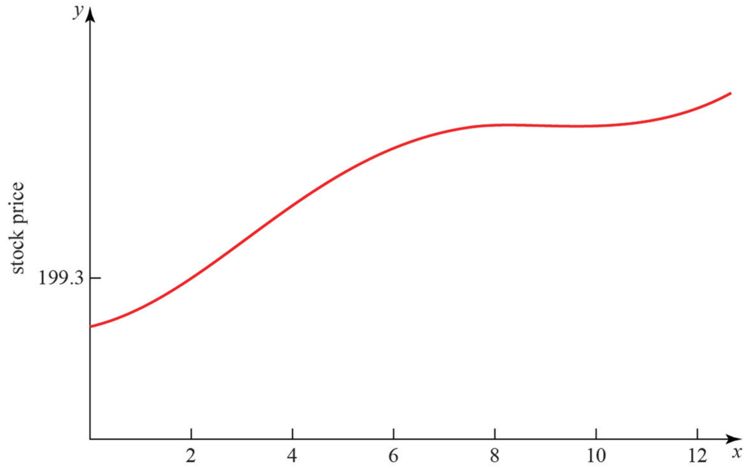
(b) Using $t_0 = 19.35$ as a first approximation to $\alpha$, apply the Newton-Raphson procedure once to $h(t)$ to find a second approximation to $\alpha$, giving your answer to 3 decimal places. [5]

(c) By considering the change of sign of $h(t)$ over an appropriate interval, determine if your answer to part (b) is correct to 3 decimal places. [3]

Total: 10

6.

$$p(t) = \frac{1}{10}\ln(t+1) - \cos\left(\frac{t}{2}\right) + \frac{1}{10}t^{\frac{3}{2}} + 199.3, \qquad 0 \le t \le 12$$




(a) Above is a graph of the price of a stock during a 12-hour trading window. The equation of the curve is given above. Show that the price reaches a local maximum in the interval $8.5 < t < 8.6$. [5]

(b) Above shows that the price reaches a local minimum between 9 and 11 hours after trading begins. Using the Newton-Raphson procedure once and taking $t_0 = 9.9$ as a first approximation, find a second approximation of when the price reaches a local minimum. [6]

Total: 11