Pearson Edexcel Level 3

GCE Mathematics 9MA0

Practice Paper F

Pure Mathematics

Time allowed: 2 **hours**

Centre:

Name:

Teacher:

Question	Points	Score
1	5	
2	4	
3	8	
4	3	
5	6	
6	6	
7	6	
8	2	
9	9	
10	5	
11	6	
12	7	
13	5	
14	4	
15	5	
16	10	
17	10	
Total:	101	

1. Show that [5]

$$\frac{6(x+7)}{(5x-1)(2x+5)}$$

can be written in the form

$$\frac{A}{5x-1}+\frac{B}{2x+5}.$$

Find the values of the constants A and B.

2. Use proof by contradiction to show that there exist no integers a and b for which $25a + 15b = 1$. [4]

3. A curve has parametric equations

$$x = \cos(2t), \quad y = \sin(t), \quad -\pi \le t \le \pi.$$

(a) Find an expression for $\mathrm{d}y/\mathrm{d}t$ in terms of t. [3]

Leave your answer as a single trigonometric ratio.

(b) Find an equation of the normal to the curve at the point A where $t = -\frac{5\pi}{6}$. [5]

Total: 8

4. Showing all steps, find [3]

$$\int \cot(3x)\,\mathrm{d}x.$$

5. A triangle has vertices $A(-2, 0, -4)$, $B(-2, 4, -6)$ and $C(3, 4, 4)$. [6]

   By considering the side lengths of the triangle, show that the triangle is a right-angled triangle.

6. The functions p and q are defined by

$$p\colon x \mapsto x^2 \quad \text{and} \quad q\colon x \mapsto 5 - 2x.$$

(a) Given that $pq(x) = qp(x)$, show that $3x^2 - 10x + 10 = 0$. [4]

(b) Explain why $3x^2 - 10x + 10 = 0$ has no real solutions. [2]

Total: 6

7. Prove by contradiction that there are infinitely many prime numbers. [6]

8. In a rainforest, the area covered by trees, F, has been measured every year since 1990. It was found that the rate of loss of trees is proportional to the remaining area covered by trees. [2]

   Write down a differential equation relating F to t, where t is the numbers of years since 1990.

9. At the beginning of each month Kath places £100 into a bank account to save for a family holiday. Each subsequent month she increases her payments by 5%. Assuming the bank account does not pay interest, find

   (a) the amount of money in the account after 9 months. [3]

   (b) Month n is the first month in which there is more than £6000 in the account. Show that [4]

$$n > \frac{\log(4)}{\log(1.05)}.$$

   (c) Maggie begins saving at the same time as Kath. She initially places £50 into the same account and plans to increase her payments by a constant amount each month. [2]

   Given that she would like to reach a total of £6000 in 29 months, by how much should Maggie increase her payments each month?

Total: 9

10. Find [5]

$$\int \cos^2(6x)\,\mathrm{d}x.$$

11. (a) Prove that [3]

$$\frac{\tan(x) - \sec(x)}{1 - \sin(x)} \equiv -\sec(x), \quad x \neq (2n+1)\frac{\pi}{2}.$$

(b) Hence solve, in the interval $0 \leq x \leq 2\pi$, the equation [3]

$$\frac{\tan(x) - \sec(x)}{1 - \sin(x)} = \sqrt{2}.$$

Total: 6

12. A large arch is planned for a football stadium. The parametric equations of the arch are

$$x = 8(t + 10), \quad y = 100 - t^2, \quad -19 \le t \le 10$$

where x and y are distances in metres. Find

(a) the cartesian equation of the arch, [3]

(b) the width of the arch, [2]

(c) the greatest possible height of the arch. [2]

Total: 7

13. [5]

$$\frac{x^3 + 8x^2 - 9x + 12}{x+6} = Ax^2 + Bx + C + \frac{D}{x+6}.$$

Find the values of the constants A, B, C and D.

14. The volume of a sphere $V\,\mathrm{cm}^3$ is related to its radius r cm by the formula $V = \frac{4}{3}\pi r^3$. The surface area of the sphere is also related to the radius by the formula $S = 4\pi r^2$. Given that the rate of decrease in surface area, in $\mathrm{cm}^2\mathrm{s}^{-1}$, is $\frac{\mathrm{d}S}{\mathrm{d}t} = -12$, find the rate of decrease of volume $\frac{\mathrm{d}V}{\mathrm{d}t}$. [4]

15. Find [5]

$$\int \sin^3(x)\, \mathrm{d}x.$$

16.

$$h(t) = 40\ln(t+1) + 40\sin\left(\frac{t}{5}\right) - \frac{1}{4}t^2, \quad t \geq 0.$$

The graph $y = h(t)$ models the height of a rocket t seconds after launch.

(a) Show that the rocket returns to the ground between 19.3 and 19.4 seconds after launch. [2]

(b) Using $t_0 = 19.35$ as a first approximation to α, apply the Newton-Raphson procedure once to $h(t)$ to find a second approximation to α, giving your answer to 3 decimal places. [5]

(c) By considering the change of sign of $h(t)$ over an appropriate interval, determine if your answer to part (b) is correct to 3 decimal places. [3]

Total: 10

17. (a) Show that in $\triangle KLM$ with $\overrightarrow{KL} = 3i + 0j - 6k$ and $\overrightarrow{LM} = 2i + 5j + 4k$, $\angle KLM = 66.4°$ to one decimal place. [7]

(b) Hence find $\angle LKM$ and $\angle LMK$. [3]

Total: 10