Pearson Edexcel Level 3

GCE Mathematics 9MA0

Practice Paper C

Pure Mathematics

Time allowed: 2 hours

Centre:

Name:

Teacher:

Question	Points	Score
1	6	
2	5	
3	6	
4	4	
5	5	
6	4	
7	5	
8	7	
9	6	
10	5	
11	6	
12	6	
13	6	
14	4	
15	13	
16	12	
Total:	100	

1. [6]

$$\frac{18x^2 - 98x + 78}{(x-4)^2(3x+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{3x+1}, \quad x > 4.$$

Find the values of the constants A, B and C.

2. A curve C has equation $4^x = 2xy$ for $x > 0$. [5]

   Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point C with coordinates $(2, 4)$.

3. (a) Show that [3]

$$\cos(7x) + \cos(3x) = 2\cos(5x)\cos(2x)$$

by expanding $\cos(5x + 2x)$ and $\cos(5x - 2x)$ using the compound-angle formulae.

(b) Hence find $\int \cos(5x)\cos(2x)\,\mathrm{d}x$. [3]

Total: 6

4. The temperature of a mug of coffee at time t can be modelled by the equation

$$T(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t},$$

where $T(t)$ is the temperature, in °C, of the coffee at time t minutes after the coffee was poured into the mug and T_R is the room temperature in °C.

(a) Using the equation for this model, explain why the initial temperature of the coffee is independent of the initial room temperature. [2]

(b) Calculate the temperature of the coffee after 10 minutes if the room temperature is 20°C. [2]

Total: 4

5. Prove by contradiction that if n is odd, $n^3 + 1$ is even. [5]

6. A curve C has parametric equation [4]

$$x = \sec^2(t) + 1, \quad y = 2\sin(t), \quad -\frac{\pi}{4} \leq t \leq \frac{\pi}{4}.$$

Show that a cartesian equation of C is $y = \sqrt{\frac{8-4x}{1-x}}$ for a suitable domain which should be stated.

7. An infinite geometric series has first four terms $1-4x+16x^2-64x^3+\ldots$. The series is convergent.

   (a) Find the set of possible values of x for which the series converges. [2]

   (b) Given that [3]

$$\sum_{r=1}^{\infty}(-4x)^{r-1}=4,$$

   calculate the value of x.

Total: 5

8.

$$f(x) = 2 - 3\sin^3(x) - \cos(x),$$

where x is in radians.

(a) Show that $f(x) = 0$ has a root α between $x = 1.9$ and $x = 2.0$. [2]

(b) Using $x_0 = 1.95$ as a first approximation, apply the Newton–Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places. [5]

Total: 7

9. Given that $(b-a)i - 2abcj + 2k = 10i - 96j + (7a + 5b)k$, find the values of a, b and c. [6]

10. Use proof by contradiction to show that there are no positive integer solutions to the statement $x^2 - y^2 = 1$. [5]

11. The function $g(x)$ is defined by $g(x) = x^2 - 8x + 7, x \in \mathbb{R}, x > 4$. [6]

Find $g^{-1}(x)$ and state its domain and range.

12. [6]

$$f(x) = \frac{4x^2 + x - 23}{(x-3)(4-x)(x+5)}, \quad x > 4.$$

Given that $f(x)$ can be expressed in the form

$$\frac{A}{x-3} + \frac{B}{4-x} + \frac{C}{x+5},$$

find the values of A, B and C.

13. The curve C has equation $y = x^3 + 6x^2 - 12x + 6$.

    (a) Show that C is concave on the interval $[-5, -3]$. [3]

    (b) Find the coordinates of the point of inflection. [3]

Total: 6

14. Find [4]

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \sin(4x)(1-\cos(4x))^3 \, \mathrm{d}x.$$

15.

$$\frac{4x^2-4x-9}{(2x+1)(x-1)} \equiv A + \frac{B}{2x+1} + \frac{C}{x-1}.$$

(a) Find the values of the constants A, B and C. [6]

(b) Hence, or otherwise, expand $\dfrac{4x^2-4x-9}{(2x+1)(x-1)}$ in ascending powers of x, as far as the x^2 term. [6]

(c) Explain why the expansion is not valid for $x = \frac{3}{4}$. [1]

Total: 13

16. A large cylindrical tank has radius 40 m. Water flows into the cylinder from a pipe at a rate of $4000\pi\ \mathrm{m^3 min^{-1}}$. At time t, the depth of water in the tank is hm. Water leaves the bottom of the tank through another pipe at a rate of $50\pi h\ \mathrm{m^3 min^{-1}}$.

    (a) Show that t minutes after water begins to flow out of the bottom of the cylinder [6]

$$160\frac{\mathrm{d}h}{\mathrm{d}t} = 400 - 5h$$

    (b) When $t = 0$min, $h = 50$m. Find the exact value of t when $h = 60$m. [6]

Total: 12