Pearson Edexcel

A Level Mathematics 9MA0

Unit Test

11 Integration – 2

Question	Points	Score
1	7	
2	11	
3	10	
4	10	
5	12	
Total:	50	

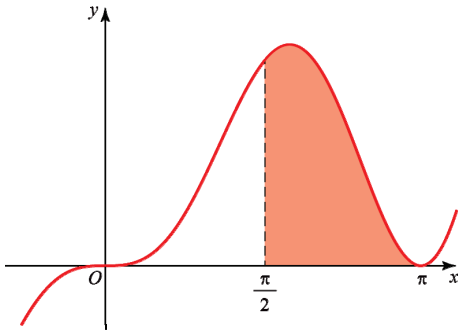
Time allowed: 50 **minutes**

School:

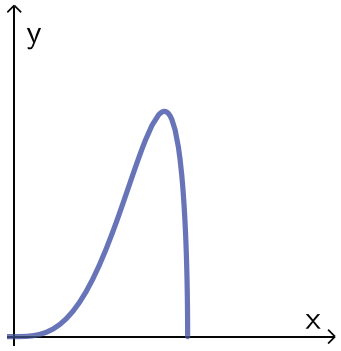
Name:

Teacher:

1. The diagram shows part of the curve with equation $y = x\sin^2(x)$. The finite region bounded by the line with equation $x = \frac{\pi}{2}$, the curve and the x-axis is shown shaded in the diagram. Find the area of the shaded region. [7]



2. The diagram shows the curve with equation $y = \frac{1}{2}x^3\sqrt{4-x^2}$.



(a) Complete the table with the value of y corresponding to $x = 1.5$. Give your answer correct to 5 decimal places. [1]

x	0	0.5	1	1.5	2
y	0	0.12103	0.86603		0

Given that

$$I = \int_0^2 \frac{1}{2}x^3\sqrt{4-x^2}\,\mathrm{d}x$$

(b) Use the trapezium rule with 4 equal width strips to find an approximate value of I, giving your answer to 4 significant figures. [3]

(c) By using an appropriate substitution, or otherwise, find the exact value of I, leaving your answer as a rational number in its simplest form. [6]

(d) Suggest one way in which your estimate using a trapezium rule could be improved. [1]

Total: 11

3.

$$f(x) = \frac{21 - 14x}{(1 - 4x)(2x + 3)}, x \neq \frac{1}{4}, x \neq -\frac{3}{2}.$$

(a) Given that [5]

$$f(x) = \frac{A}{1 - 4x} + \frac{B}{2x + 3},$$

find the values of the constants A and B.

(b) Find the exact value of $\int_{-1}^{0} f(x)\,\mathrm{d}x$. [5]

Total: 10

4. The value of a computer, V, decreases over time, t, measured in years. The rate of decrease of the value is proportional to the remaining value.

   (a) Given that the initial value of the computer is V_0, show that [4]

$$V = V_0 e^{-kt}.$$

   After 10 years the value of the computer is $\frac{1}{5}V_0$.

   (b) Find the exact value of k. [3]

   (c) How old is the computer when its value is only 5% of its original value? Give your answer to 3 significant figures. [3]

Total: 10

5. A large cylindrical tank has radius 40 m. Water flows into the cylinder from a pipe at a rate of $4000\pi \text{m}^3\text{min}^{-1}$. At time t, the depth of water in the tank is hm. Water leaves the bottom of the tank through another pipe at a rate of $50\pi h \text{m}^3\text{min}^{-1}$.

   (a) Show that t minutes after water begins to flow out of the bottom of the cylinder, [6]

$$160\frac{\text{d}h}{\text{d}t} = 400 - 5h$$

   (b) When $t = 0$ min, $h = 50$m. [6]

   Find the exact value of t when $h = 60$m.

Total: 12