Pearson Edexcel Level 3

GCE Mathematics 9MA0

Practice Paper E

Pure Mathematics

Time allowed: 2 **hours**

Centre:

Name:

Teacher:

Question	Points	Score
1	3	
2	5	
3	9	
4	5	
5	6	
6	8	
7	7	
8	9	
9	4	
10	4	
11	9	
12	10	
13	10	
14	11	
Total:	100	

1. Prove by exhaustion that [3]

$$1 + 2 + 3 + \ldots + n \equiv \frac{n(n+1)}{2}$$

for positive integers from 1 to 6 inclusive.

2. (a) When θ is small, show that the equation $\dfrac{1+\sin(\theta)+\tan(2\theta)}{2\cos(3\theta)-1}$ can be written as $\dfrac{1}{1-3\theta}$. [4]

   (b) Hence write down the value of $\dfrac{1+\sin(\theta)+\tan(2\theta)}{2\cos(3\theta)-1}$ when θ is small. [1]

Total: 5

3. A stone is thrown from the top of a building. The path of the stone can be modelled using the parametric equations $x = 10t, y = 8t - 4.9t^2 + 10, t \geq 0$, where x is the horizontal distance from the building in metres and y is the vertical height of the stone above the level ground in metres.

   (a) Find the horizontal distance the stone travels before hitting the ground. [4]

   (b) Find the greatest vertical height. [5]

Total: 9

4. Given that $x = \sec(4y)$, find

   (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. [2]

   (b) Show that [3]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{x\sqrt{x^2 - 1}}$$

   where k is a constant which should be found.

Total: 5

5.

$$f(x) = \frac{6}{x} + \frac{3}{x^2} - 7x^{\frac{5}{2}}.$$

(a) Find $\int f(x)\,\mathrm{d}x$. [3]

(b) Evaluate [3]

$$\int_4^9 f(x)\,\mathrm{d}x,$$

giving your answer in the form $m + n\ln(p)$, where m, n and p are rational numbers.

Total: 6

6. Figure 1 shows a sketch of part of the graph $y = f(x)$ where $f(x) = 3|x - 4| - 5$.

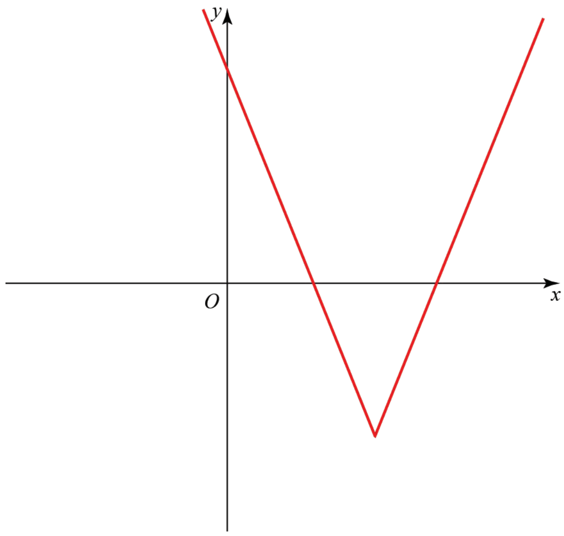

Figure 1:

(a) State the range of f. [1]

(b) Given that $f(x) = -\frac{1}{3}x + k$, where k is a constant has two distinct roots, state the possible values of k. [7]

Total: 8

7. [7]

$$f(x) \equiv \frac{9x^2 + 25x + 16}{9x^2 - 16}$$

Show that $f(x)$ can be written in the form

$$A + \frac{B}{3x-4} + \frac{C}{3x+4},$$

where A, B and C are constants to be found.

8. A ball is dropped from a height of 80 cm. After each bounce it rebounds to 70% of its previous maximum height.

   (a) Write a recurrence relation to model the maximum height in centimetres of the ball after each subsequent bounce. [2]

   (b) Find the height to which the ball will rebound after the fifth bounce. [2]

   (c) Find the total vertical distance travelled by the ball before it stops bouncing. [4]

   (d) State one limitation with the model. [1]

Total: 9

9. Solve [4]

$$6\sin(\theta + 60) = 8\sqrt{3}\cos(\theta)$$

in the range $0 \leq \theta \leq 360°$. Round your answer to 1 decimal place.

10. Use proof by contradiction to show that there is no greatest positive rational number. [4]

11. The first three terms in the binomial expansion of $(a + bx)^{\frac{1}{3}}$ are $4 - \frac{1}{8}x + cx^2 + \ldots$.

 (a) Find the values of a and b. [5]

 (b) State the range of values of x for which the expansion is valid. [2]

 (c) Find the value of c. [2]

Total: 9

12. The diagram shows a cuboid whose vertices are O, A, B, C, D, E, F and G. a, b and c are the vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively. The points M and N lie on $\overrightarrow{OA}$ such that $OM : MN : NA = 1 : 2 : 1$. The points K and L lie on EF such that $EK : KL : LF = 1 : 2 : 1$. [10]

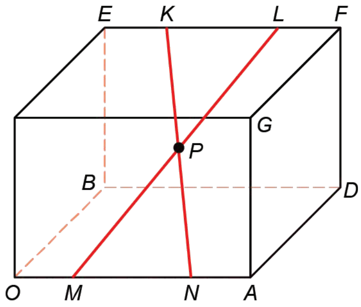

Figure 2:

Prove that the diagonals KN and ML bisect each other at P.

13. The value of a computer, V, decreases over time, t, measured in years.

The rate of decrease of the value is proportional to the remaining value.

Given that the initial value of the computer is V_0,

(a) Show that $V = V_0 e^{-kt}$. [4]

(b) After 10 years the value of the computer is $\frac{1}{5}V_0$. Find the exact value of k. [3]

(c) How old is the computer when its value is only 5% of its original value? [3]

Give your answer to 3 significant figures.

Total: 10

14.

$$p(t) = \frac{1}{10}\ln(t+1) - \cos\left(\frac{t}{2}\right) + \frac{1}{10}t^{\frac{3}{2}} + 199.3, \quad 0 \le t \le 12.$$

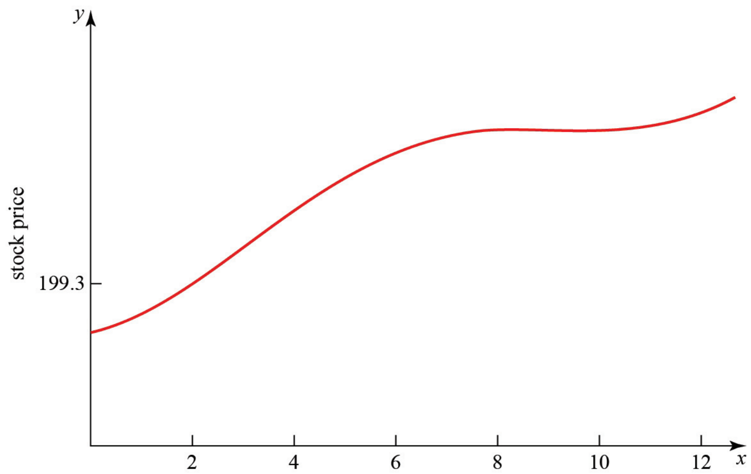

Figure 3:

Figure 3 is a graph of the price of a stock during a 12-hour trading window. The equation of the curve is given above.

(a) Show that the price reaches a local maximum in the interval $8.5 < t < 8.6$. [5]

(b) Figure 3 shows that the price reaches a local minimum between 9 and 11 hours after trading begins. Using the Newton–Raphson procedure once and taking $t_0 = 9.9$ as a first approximation, find a second approximation of when the price reaches a local minimum. [6]

Total: 11

(Q14 contined)