Pearson Edexcel Level 3

GCE Mathematics 9MA0

Practice Paper A

Pure Mathematics

Time allowed: 2 hours

Centre:

Name:

Teacher:

Question	Points	Score
1	4	
2	3	
3	5	
4	4	
5	7	
6	8	
7	8	
8	8	
9	9	
10	4	
11	10	
12	7	
13	12	
14	4	
15	7	
Total:	100	

1. It is suggested that the sequence $a_k = 2^k + 1, k \dots, n$ produces only prime numbers.

   (a) Show that a_1, a_2 and a_4 produce prime numbers. [2]

   (b) Prove by counter example that the sequence does not always produce a prime number. [2]

Total: 4

2. Find the angle that the vector $a = 4i - j + 3k$ makes with the positive y-axis. [3]

3.

$$g(x) = 3\sin\left(\left(\frac{x}{6}\right)^3\right) - \frac{1}{10}x - 1, \quad -40 < x < 20,$$

x is in radians.

(a) Show that the equation $g(x) = 0$ can be written as [3]

$$x = 6\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)}.$$

(b) Using the formula [2]

$$x_{n+1} = 6\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x_n\right)},$$

$x_0 = 4$, find, to 3 decimal places, the value of x_1, x_2 and x_3.

Total: 5

4. The first 3 terms of a geometric sequence are [4]

$$k+2, 4k, 2k^2, k>0.$$

Find the value of k.

5. [7]

$$f(x) = \frac{x^4 + 2x^3 - 29x^2 - 47x + 77}{x^2 - 2x - 15}$$

Show that $f(x)$ can be written as

$$Px^2 + Qx + R + \frac{V}{x+3} + \frac{W}{x-5}$$

and find the values of P, Q, R, V and W.

6. Figure 1 shows a logo comprised of a rhombus surrounded by two arcs. Arc BAD has centre C and arc BCD has centre A. [8]

Some of the dimensions of the logo are shown in the diagram.

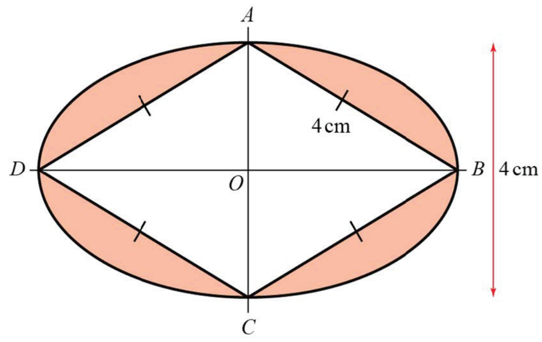

Figure 1:

Prove that the shaded area of the logo is $\frac{2}{3}\left(16\pi - 24\sqrt{3}\right)$.

7. C has parametric equations

$$x = \frac{1+4t}{1-t}, \quad y = \frac{2+bt}{1-t}, \quad -1 \le t \le 0.$$

(a) Show that the cartesian equation of C is [4]

$$y = \left(\frac{2+b}{5}\right)x + \frac{8-b}{5},$$

over an appropriate domain.

(b) Given that C is a line segment and that the gradient of the line is -1, show that the length of the line segment is $a\sqrt{2}$, where a is a rational number to be found. [4]

Total: 8

8. A toy soldier is connected to a parachute. The soldier is thrown into the air from ground level.

   The height, in metres, of the soldier above the ground can be modelled by the equation

$$H = \frac{4t^{\frac{2}{3}}}{t^2+1}, \quad 0 \leq t \leq 6s,$$

   where H is height of the soldier above the ground and t is the time since the soldier was thrown.

   (a) Show that [4]

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8(1-2t^2)}{3\sqrt[3]{t}(t^2+1)^2}.$$

   (b) Using the differentiated function, explain whether the soldier was increasing or decreasing in height after 2 seconds. [2]

   (c) Find the exact time when the soldier reaches a maximum height. [2]

Total: 8

9. (a) Show that [4]

$$\tan^4(x) = \sec^2(x)\tan^2(x) + 1 - \sec^2(x).$$

(b) Hence find the exact value of [5]

$$\int_0^{4\pi} \tan^4(x)\,\mathrm{d}x.$$

Total: 9

10. Use proof by contradiction to show that, given a rational number a and an irrational number b, $a - b$ is irrational. [4]

11.

$$f(x) = |2x + 3| - 4, \quad x \in \mathbb{R}.$$

(a) Sketch the graph of $y = f(x)$, labelling its vertex and any points of intersection with the coordinate axes. [5]

(b) Find the coordinates of the points of intersection of $y = |2x + 3| - 4$ and $y = -\frac{1}{4}x + 2$. [5]

Total: 10

12. (a) Prove that $(\sin(3\theta) + \cos(3\theta))^2 = 1 + \sin(6\theta)$. [3]

(b) Use the result to solve, for $0 \leq \theta \leq \frac{\pi}{2}$,the equation [4]

$$\sin(3\theta) + \cos(3\theta) = \sqrt{\frac{2 + \sqrt{2}}{2}}.$$

Give your answer in terms of π. Check for extraneous solutions.

Total: 7

13.

$$f(x) = \frac{6}{2+3x} - \frac{4}{3-5x}, \quad |x| < \frac{3}{5}.$$

(a) Show that the first three terms in the series expansion of $f(x)$ can be written as [7]

$$\frac{5}{3} - \frac{121}{18}x + \frac{329}{108}x^2.$$

(b) Find the exact value of $f(0.01)$. Round your answer to 7 decimal places. [2]

(c) Find the percentage error made in using the series expansion in part (a) to estimate the value of $f(0.01)$. [3]

Give your answer to 2 significant figures.

Total: 12

14. Jacob is making some patterns out of squares. The first 3 patterns in the sequence are shown in Figure 2.

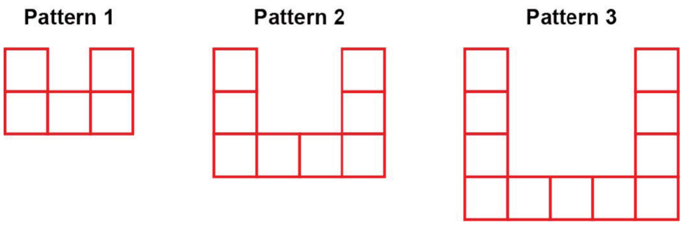

Figure 2:

(a) Find an expression, in terms of n, for the number of squares required to make pattern n. [2]

(b) Jacob uses a total of 948 squares in constructing the first k patterns. [2]

Show that $3k^2 + 7k - 1896 = 0$.

Total: 4

15. Figure 3 shows part of the curve with equation $y = x\sin^2(x)$. The finite region bounded by the line with equation $x = \frac{\pi}{2}$, the curve and the x-axis is shown shaded in the diagram. [7]

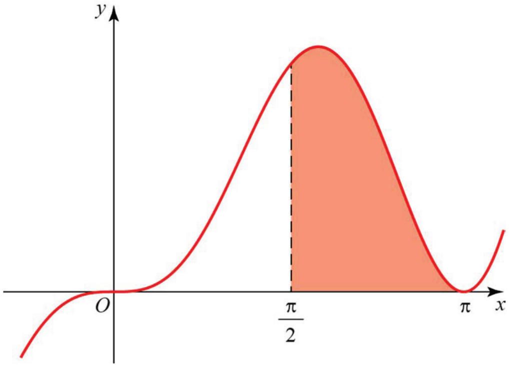

Figure 3:

Find the area of the shaded region.